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DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

GEOMETRY.

383. Proposed by S. A. COREY, Hiteman, Iowa.

Let $ABCDE$ be a pentagon, plane, or gauche, with sides AB, BC, CD, and DE, of lengths, w, x, y, and z, respectively. Construct four other pentagons, $AB_1C_1D_1E_1$, $AB_{11}C_{11}D_{11}E_{11}$, $AB_{111}C_{111}D_{111}E_{111}$, and $AB_{iv}C_{iv}D_{iv}E_{iv}$, having a common vertex at A, and with four consecutive sides in each parallel to the corresponding consecutive sides, AB, BC, CD, and DE, in $ABCDE$. Further, let the lengths of the sides AB_1, B_1C_1, C_1D_1, D_1E_1, AB_{11}, $B_{11}C_{11}$, $C_{11}D_{11}$, $D_{11}E_{11}$, AB_{111}, $B_{111}C_{111}$, $C_{111}D_{111}$, $D_{111}E_{111}$, AB_{iv}, $B_{iv}C_{iv}$, $C_{iv}D_{iv}$, $D_{iy}E_{iv}$, be $-wW$, xX, yY, zZ, wX, xW, yZ, $-zY$, wY, yW, zX, $-xZ$, wZ, zW, xY, and $-yX$, respectively; the minus sign indicating the reversal of direction of the corresponding side. Prove that $(W^2+X^2+Y^2+Z^2)(w^2+x^2+y^2+z^2)=AE_1^2+AE_{11}^2+AE_{111}^2+AE_{iv}^2$.

Solution by the PROPOSER.

Let $\mathbf{w}$, $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ represent the vector sides, AB, BC, CD, and DE, respectively, of the pentagon $ABCDE$, the tensors of corresponding vectors being w, x, y, and z. Then in the other four pentagons will $-W\mathbf{w}$, $X\mathbf{x}$, $Y\mathbf{y}$, $Z\mathbf{z}$, $X\mathbf{w}$, $W\mathbf{x}$, $Z\mathbf{y}$, $-Y\mathbf{z}$, $Y\mathbf{w}$, $W\mathbf{y}$, $X\mathbf{z}$, $-Z\mathbf{x}$, $Z\mathbf{w}$, $W\mathbf{z}$, $Y\mathbf{x}$, and $-Y\mathbf{y}$, represent the vector sides AB_1, B_1C_1, C_1D_1, D_1E_1, AB_{11}, $B_{11}C_{11}$, $C_{11}D_{11}$, $D_{11}E_{11}$, AB_{111}, $B_{111}C_{111}$, $C_{111}D_{111}$, $D_{111}E_{111}$, AB_{iv}, $B_{iv}C_{iv}$, $C_{iv}D_{iv}$, and $D_{iv}E_{iv}$, respectively. The remaining or fifth side of each of the four pentagons may be represented as the sum of the other four vector sides, whence

$$AE_1=-W\mathbf{w}+X\mathbf{x}+Y\mathbf{y}+Z\mathbf{z},\quad AE_{11}=X\mathbf{w}+W\mathbf{x}+Z\mathbf{y}-Y\mathbf{z},$$

$$AE_{111}=Y\mathbf{w}+W\mathbf{y}+X\mathbf{z}-Z\mathbf{x},\quad AE_{iv}=Z\mathbf{w}+W\mathbf{z}+Y\mathbf{x}-X\mathbf{y}.$$

Substituting the squares of these four vector sums for the squares of the fifth sides which constitute the second member of the given equation, expanding and adding, their sum is found to be identical with the first member, due regard being had to the change of signs. Q. E. D.

The foregoing non-geometric solution is given to show the advantage sometimes resulting from the introduction of vectors in the handling of certain classes of geometric problems. In this case the remarkable simplicity and directness of the solution is due to the fact that the equation given in the problem is a geometric interpretation of the algebraic identity,

$$(W^2+X^2+Y^2+Z^2)(w^2+x^2+y^2+z^2)=(-wW+xX+yY+zZ)^2$$
$$+(wX+xW+yZ-zX)^2+(wY+yW+zX-xZ)^2+(wZ+zW+xY-yX)^2.$$

We may observe that every algebraic identity, in which each term is of the second degree in so far as certain letters are concerned, may be given a geometric interpretation if each of such letters be used to represent a certain vector, and if the scalar but not the vector portion of the product be employed in the interpretation. That this is true follows from the fact that the non-commutative character of vector multiplication does not alter or affect the scalar portion of the product, if each term of such product contains either the product of two separate vectors or the square of some one vector; *i. e.*, if no term in the expanded form is of a degree higher or lower than the second in the letters used to designate vectors.

A solution similar to the foregoing may be employed in problem 377.

384. Proposed by S. LEFSEHETZ, Clark University.

Let ABC be a triangle, O a circle tangent to its three sides, T a variable tangent of O, which cuts the sides BC, CA, AB in a, b, c. Oa', Ob', Oc' the perpendiculars in O to Oa, Ob, Oc, cutting, respectively, T in points a', b', c'. Prove that Aa', Bb', Cc' meet in a point t, and find the locus of t when T varies. Purely geometrical proofs wanted.

No solution of this problem has been received.

385. Proposed by V. M. SPUNAR, M. and E. E., Chicago, Ill.

Given a triangle ABC, find the radius of a circle touching two of its sides and a line parallel to the third, at a distance $d=u+2r$.

Solution by A. H. HOLMES, Brunswick, Maine.

Let a, b, and c be the sides of the given triangle, c the base. Then h=altitude of the triangle$=\dfrac{\sqrt{[4a^2c^2-(a^2-b^2+c^2)]}}{2c}$, and R=radius of the inscribed circle $=\dfrac{\sqrt{[4a^2c^2-(a^2-b^2+c^2)]}}{2(a+b+c)}$.

Put r=radius of circle touching a and b and a line parallel to c at a distance from c, $2r+u$. Then $h:R=h-(2r+u):r$.

$$\therefore r=\frac{(h-u)R}{h+2R}.$$

Putting for h and R their values in terms of a, b, and c, we have,

$$r=\frac{\sqrt{[4a^2c^2-(a^2-b^2+c^2)^2]}-u}{a+b+3c}.$$

CALCULUS.

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306. Proposed by FRANCIS RUST, C. E., Pittsburg, Pa.

Express in elliptic integrals: $A_\theta=\displaystyle\int_0^\theta \frac{dx}{\sqrt{(1-x^4)}}$; $0<\theta<1$.